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Abstract

This paper develops a simple framework to analyze the links between corruption and the unofficial economy and their implications for the official economy. In a model of self-selection with heterogeneous entrepreneurs, we show that the entrepreneurs' option to flee to the underground economy constrains a corrupt official's ability to introduce distortions to the economy for private gains. The unofficial economy thus mitigates government-induced distortions and, as a result, leads to enhanced economic activities in the official sector. In this sense, the presence of the unofficial sector acts as a complement to the official economy rather than a substitute.

Keywords: corruption, shadow economy, official economy, self-selection.

JEL Classification: D9, H2, K4, L1.

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1. Introduction

The importance of the shadow economy has been well documented, especially for many developing countries and economies in transition where legal and political institutions are not adequate to support efficient market activities [see de Soto (1989)].[1] By definition, the unofficial economy constitutes activities that are not recorded in the government statistics. As such, the extent of the unofficial economy in a given country is hard to measure precisely. Nonetheless, many scholars have attempted to indirectly estimate the share of the unofficial economy in GDP and have come up with a significant size of the shadow economy [see Schneider and Enste (2000) for an excellent survey on this issue]. For instance, according to Schneider and Enste's estimates based on the physical input (electricity) and currency demand, the underground sector in Nigeria, Egypt, and Thailand represents in each case nearly three-quarters the size of officially recorded GDP as averaged over 1990-93.[2] The corresponding figures for OECD countries range from 8 to 30 percent.

Despite universal recognition for the importance of the unofficial economy, its role in terms of allocation of resources and market performance is less well understood and entails considerable disagreement. As emphasized by Schneider and Enste (2000), the size, causes, and consequences of the shadow economy vary for different types of countries. In this paper, we analyze the role of the unofficial economy in a specific situation where entrepreneurs are required to purchase a license from a *corrupt* official to open a shop in the official economy, as in Shleifer and Vishny [1993]. We consider such a situation as an environment for studying the unofficial economy because one of the main reasons for the very existence of the unofficial sector is entrepreneurs' attempts to find a shelter from government-induced distortions such as excessive taxes, regulation, and graft.[3] In such a framework, we ask the

[1] Shadow economy has also been called the underground economy, unofficial economy, hidden economy or informal economy. We will use all these terms interchangeably.

[2] Kaufmann and Kaliberda (1996), by proxying the overall economic activity with electricity consumption, also report the following figures for transition economies as shares of unofficial activity in total activity for 1989: Bulgaria, 22.8 percent; Czech Republic and Slovakia, 6.0 percent; Hungary, 27.0 percent; Poland 15.7 percent; Romania 22.3 percent; the former Soviet republics, 12.0 percent. Another estimate by 1996 Russia's Central Statistical Office, Goskomstat, indicates that the unofficial economy is about 20 percent of total activity; see Johnson, Kaufman and Shleifer (1997).

[3] Friedman et al. (1999) argue that entrepreneurs go underground not to avoid official taxes but to reduce the burden of bureaucracy and corruption. They draw this conclusion in their cross-country analysis that higher tax rates are associated with less unofficial activity as a percent of GDP. High correlation between corruption and

question of whether the unofficial economy is a substitute or a complement to the official economy. In other words, we examine whether economic activities in the unofficial economy crowd out or promote economic activities in the official sector when the government official uses the official sector as the source for his/her private gains.

We show that the entrepreneurs' option to flee to the underground sector constrains the corrupt official's ability to introduce distortion to the economy for private gains. The unofficial economy thus mitigates government-induced distortions and, as a result, leads to enhanced economic activities in the official sector. In this sense, the presence of the unofficial sector plays as a complement to the official economy rather than a substitute.[4]

Levenson and Maloney (1996) also develop a model where the underground economy arises endogenously due to self-selection of heterogeneous entrepreneurs.[5] Running a business in the formal economy generates several benefits from publicly provided goods but also creates fixed costs for the entrepreneurs as they have to comply with regulatory measures (e.g. reporting requirements). Entrepreneurs are heterogeneous in their abilities to manage their firms. Over time, entrepreneurs learn more about their abilities. Entrepreneurs typically start their businesses in the informal sector to avoid the high compliance costs. Once a firm is sufficiently large (because the entrepreneur is of high ability), the firm switches to the official economy to benefit from the publicly provided goods that are complementary inputs in production. Levenson and Maloney thus model the dynamic transition process between the formal and informal sectors.

Johnson, Kaufmann, and Shleifer (1997) present a model of full employment where the labor can be employed either in the official or unofficial sectors. In this model, the expansion of the informal sector always implies the contraction of the formal sector and vice versa. In our model, the extent of the total economic activities is endogeneously determined

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underground economic activities has been well established in empirical research. See Johnson, Kaufmann and Zoido-Lobaton (1998).

[4] Schneider and Enste (2000) point out that at least two thirds of the income earned in the shadow economy is immediately spent in the official economy and thus can have a positive effect on the official economy. The mechanism through which the shadow economy has a positive effect on the official economy in our paper is thus different from that in Schneider and Enste (2000).

[5] See also Ahlin (2001) for a model of occupational choice with heterogeneous entrepreneurs in the context of corruption.

by government policies. In contrast to Johnson *et al.*, we show that the existence of the shadow economy helps the expansion of the official sector. In addition, our model has a unique equilibrium whereas their model exhibits multiple equilibria due to increasing returns to sector size and due to path-dependence in the equilibrium selection.

Elsewhere in Choi and Thum (1998, forthcoming), we adopt a similar framework to analyze the dynamics of corruption, but we do not consider the possibility of the underground economy.

The remainder of the paper is organized in the following way. Section 2 sets up the basic model and shows how corruption and the shadow economy interact when entrepreneurs have the option of operating in the shadow economy to avoid tax/regulatory burdens imposed by the government. We characterize the pattern of self-selection by entrepreneurs and the optimal monetary demand schedule for the official. In Section 3, we extend the basic model to allow for an endogenous choice of the capital stock by entrepreneurs. As the detection probability typically rises with the size of illegally operating firms, evasion into the shadow economy comes at the cost of an inefficiently small capital stock. Section 4 considers the case where the corrupt authority not only demands bribery payments but also provides useful public goods for production. The existence of an unofficial sector creates inefficiently small incentives for public good provision. Section 5 contains concluding remarks.

2. A Simple Model of Corruption and the Shadow Economy

Before analyzing more complex linkages between corruption and the shadow economy, we first develop the simplest feasible model of an economy with corruption where firms have the outside option of operating in the shadow economy. There is a population of entrepreneurs whose total number is normalized to unity. Entrepreneurs are heterogeneous in their ability to generate income. Let v denote an entrepreneur's gross earnings reflecting his abilities. The distribution of abilities is given by the *inverse* cumulative distribution function $F(v)$ with continuous density $F'(v) \leq 0$, that is, $F(v)$ denotes the proportion of entrepreneurs who can generate income *more* than v.

As in Shleifer and Vishny (1993), we consider the sale of government property (entry permit) by government officials as the prototype of corruption activities.[6] This is in accordance with Tanzi's (1998) statement that "the most popular and simplest definition of corruption is that it is the abuse of public power for private benefit."[7] Thus, entrepreneurs are required to make payments m to a corrupt government official as a licensee fee in order to open a shop. In addition, an entrepreneur who wants to start a business has to employ the amount of capital k. For the time being, we assume that the required capital for entry k is fixed.[8] The official sets the price of the license m to maximize his (private) revenues from licensing. As the entrepreneur's ability v is private information, the corrupt official cannot price discriminate and charges a uniform license fee m.[9] From the entrepreneurs' point of view, the bribery payment is like an additional cost of entry to the market.

Corruption without the Shadow Economy

As a benchmark case, we first consider a situation where the option to operate in the shadow economy is not available, that is, the entrepreneurs' only choice is whether to enter or not. We normalize the entrepreneurs' payoffs from not entering the market to zero. Then, entrepreneurs who can generate non-negative incomes enter the market with capital k and make the bribery payment m to the corrupt official: $\pi_{OE} = v - k - m \geq 0$. Given the official's demands m for the license, the marginal type who is indifferent between entry and exit is given by $v=k+m$.

The entry behavior of the entrepreneurs implies that the corrupt official maximizes his revenue:

$$\max_{m} \; R(m) = m \cdot F(k+m).$$

[6] See Bardhan (1997) for an excellent survey on corruption.
[7] As pointed out by Stigler (1971), "[t]he state has one basic resource which in pure principle is not shared with even the mightiest of its citizens: the power to coerce." The state's monopoly on coercion can lead to the abuse of power when public officials have wide discretion and little accountability due to the lack of formal checks and balances [World Bank (1997)].
[8] In the next section, we endogenize the amount of capital that represents the scale of operation for each entrepreneur.
[9] In a dynamic model of corruption, the official may be able to price discriminate based on the entry decisions made in the earlier period. Elsewhere in Choi and Thum (forthcoming), we analyze such a model and show that the official's *ex post* incentive to price discriminate entails the ratchet effect in that entrepreneurs have the incentive to delay entry into the market in order to receive a discount in the permit price later. However, the paper does not address the issue of the shadow economy.

Since the official's demand schedule m is uniquely determined by v, we will find it more convenient to treat v as the control variable:

$$\max_{v} \; R(v) = (v-k)\cdot F(v).$$

The marginal entrant v^* that maximizes the corrupt official's revenue is implicitly given by the first order condition:

$$F(v^*) + (v^* - k)\cdot F'(v^*) = 0. \tag{1}$$

We make the standard assumption that the distribution of types satisfies the monotone hazard rate condition, that is, $-F'/F$ is increasing:

$$-F''F + (F')^2 > 0. \tag{2}$$

This assumption ensures that the official's objective function is quasi-concave and the second order condition for the maximization problem is satisfied:

$$2\cdot F'(v) + (v-k)\cdot F''(v) < 0.^{10} \tag{3}$$

Then, the number of entrants is given by $F(v^*)$. The official demands $m^* = v^* - k$ for the license.

In contrast, the first best entry configuration can be derived by

$$\max_{v} \; W(v) = \int_{v}^{\infty} (x-k)\,dF(x). \tag{4}$$

Thus, the marginal entrant that maximizes social welfare is $v^{FB} = k$, that is, any entrepreneur who can generate more revenue than the capital cost of k should enter the market. When we evaluate the first order condition for the official's revenue maximization problem at $v^{FB} = k$,

$$F(v^{FB}) + (v^{FB} - k)\cdot F'(v^{FB}) = F(k) > 0.$$

This implies that $v^* > v^{FB} = k$. Corruption induces exit of the entrepreneurs whose types belong to $[v^{FB} = k,\ v^*)$ and hence reduces the number of entrants $F(v^*)$ below the first best level $F(v^{FB}) = F(k)$. Our simple model of corruption generates the standard result in the

[10] Using the first order condition, we can rewrite the second order condition as $2\cdot F'(v) - F''(v)\cdot F(v)/F'(v) < 0$. The second order condition holds if the distribution F satisfies the monotone hazard rate condition. This condition is a standard assumption in the incentive literature and is satisfied by most widely used distributions; see Fudenberg and Tirole [1991, p. 267].

literature [see, e.g., Rose-Ackerman (1978), Shleifer and Vishny (1993) or Choi and Thum (forthcoming)] that corruption is detrimental to welfare by reducing entry of firms. The deadweight loss due to corruption is given by $-\int_k^{v^*}(x-k)dF(x)$.

Corruption with the Shadow Economy

Production in the official economy requires a license that is costly to entrepreneurs as corrupt officials exercise their monopoly power. However, entrepreneurs do not only have the decision of whether to make the bribery payment or stay out of business. They also have the option to produce without a license in the unofficial or shadow economy.[11] This saves them the costs of bribing the corrupt official but bears the cost of possible detection and subsequent punishment.

Suppose that the probability of detection for an entrepreneur is μ, which depends on the monitoring effort of the supervising institution. For now we take this monitoring effort as given. In case of detection, we simply assume that the entrepreneur loses everything, i.e. the entire capital of the firm is confiscated as punishment. The profit of a (risk-neutral) entrepreneur in the shadow economy is then given by $\pi_{SE} = (1-\mu)\cdot v - k$.

In order to have a meaningful analysis of the shadow economy, we restrict our attention to the parameter regions in which the shadow economy constraint is binding, that is,

$$\frac{k}{1-\mu} < v^*,$$

where v^* is defined by (1).[12] This condition is satisfied if the probability of detection μ is not too high.

When entrepreneurs make their entry decisions, they choose the sector that yields the highest expected profit (π_{OE}, π_{SE}, 0). For a given bribery demand *m*, the corrupt official faces the following entry configuration:

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[11] According to Rose-Ackermann (1997), "going underground is a substitute for bribery."

[12] If the presence of the shadow economy does not constrain the corrupt official's maximization problem, the marginal type of entrepreneurs is given by v^* as defined by (1). The marginal type v^* earns zero profit when he enters the official economy. If he enters the shadow economy instead, he will earn $(1-\mu)\, v^*-k$. If $k/(1-\mu) < v^*$, the payoff from entering the shadow economy exceeds the payoffs from other options. Thus, the presence of the shadow economy serves as a binding constraint for the official's revenue maximization problem.

$$\frac{m}{\mu} \leq v \qquad \text{entry in the formal economy}$$

$$\frac{k}{1-\mu} \leq v < \frac{m}{\mu} \qquad \text{entry in the shadow economy}$$

$$v < \frac{k}{1-\mu} \qquad \text{no entry}$$

The corrupt official takes into account that potential entrants may evade his bribery demands by setting up their firms in the unofficial sector. The corrupt official, therefore, maximizes

$$\max_m \ \tilde{R}(m) = m \cdot F\left(\frac{m}{\mu}\right).$$

Once again, we treat the marginal entrant type $v = m/\mu$ as the control variable:

$$\max_v \ \tilde{R}(v) = \mu \cdot v \cdot F(v).$$

The first order condition

$$F(v) + v \cdot F'(v) = 0. \tag{5}$$

determines the marginal type of entrepreneur $\tilde{v}$ entering the official economy when operating in the shadow economy is the outside option.[13] Entrepreneurs with low abilities ($v < k/(1-\mu)$) stay out of business [see Figure 1]. Those with intermediate abilities ($k/(1-\mu) \leq v < \tilde{v}$) operate in the shadow economy. High-ability entrepreneurs ($\tilde{v} \leq v$) enter the official economy and make their contributions to the corrupt official.

The most interesting question here is how the existence of the unofficial sector influences the behavior of the corrupt official. There is a clear-cut answer to this question:

Proposition 1. In an economy with corruption, the official sector is larger when production in the unofficial sector is feasible, i.e. $\tilde{v} < v^*$. Thus, the activities in the shadow economy are complementary to the ones in the formal economy.

Proof. Evaluate (5) at v^*, which is the marginal type of entrepreneur when no unofficial sector exists: $F(v^*) + v^* \cdot F'(v^*) = k \cdot F'(v^*) < 0$ [cf. (1)] and we have $\partial R(v^*)/\partial v < 0$. Hence, $\tilde{v} < v^*$.

[13] Variables corresponding to the case where the option of fleeing to the shadow economy is possible are denoted by a tilde.

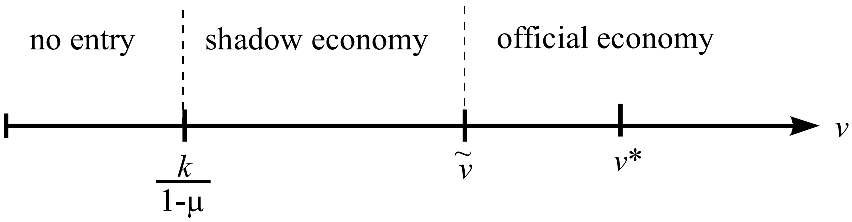

Figure 1.

An immediate corollary of Proposition 1 is that social welfare is improved in the presence of the shadow economy. There are two channels through which social welfare increases. First, the economy benefits directly from the unofficial sector because it allows entrepreneurs to produce who otherwise would not have entered at all. The direct benefits from the shadow economy are given by $\int_{k/(1-\mu)}^{\tilde{v}} (x-k)dF(x)$. Second, there are indirect benefits due to the existence of the shadow economy. Since it improves the outside option for entrepreneurs, the corrupt official is forced to *lower* his bribery demands, which leads to more entry into the official sector. This endogeneity of the official's bribery demand is crucial in understanding the complementarity between the shadow economy and the official sector.[14] As the bribery payment is a mere redistribution from the entrepreneurs to the corrupt official, any additional activity is beneficial. The indirect benefits can be represented by $\int_{\tilde{v}}^{v^*} (x-k)dF(x)$. Thus from a welfare point of view, the existence of the unofficial sector is desirable.

This result is in sharp contrast to the negative views of the shadow economy portrayed by Johnson, Kaufman, and Shleifer (1997), Levenson and Maloney (1996), and Loayza (1996) among others. In their models, the government imposes taxes on the official sector and the tax revenues collected provide public goods that increase the productivity of firms in the economy. Thus, the movement of production into the shadow economy has harmful consequences for the economy since firms in the shadow economy escape taxation and consequently the ability of the government to provide public goods.[15]

[14] If the official demanded the *same* bribery demand as in the absence of the shadow economy, then the official sector would shrink since the marginal type v^* would prefer to enter the shadow economy.
[15] In section 4, we consider the public finance aspects of the shadow economy as in Johnson et al (1997).

3. Endogenous Investment and Monitoring

So far the entrepreneurs' only choice was which sector to enter. The capital stock only played a role in deciding whether to enter the market at all. However, there are significant differences in the firms' capital stock between the official economy and the shadow economy. We will incorporate this aspect now by endogenizing the choice on the amount of capital employed in a firm.

In order to avoid being detected, firms in the shadow economy typically scale down the size their operations. This incentive to avoid detection thus prevents firms in the shadow economy from achieving economies of scale. To accommodate the phenomenon of *size dualism* observed in the empirical literature on the shadow economy [Banerjee, 1983], we modify our basic model in two aspects.[16] First, revenues are correlated positively with the capital stock. Second, the detection probability for illegal firms increases in the size of investment since larger and more capital-intensive firms are easier to detect.

Let the entrepreneur's net income in the official sector now be $v \cdot k - k = (v-1) \cdot k$ with $k \in [0, K]$.[17] The larger the capital stock and the higher the ability, the more revenue can be generated by an entrepreneur. Without corruption, the optimal decision of an entrepreneur is to enter the market and employ the maximum amount of capital K if his ability v exceeds unity ($v \geq 1$). Entrepreneurs with lower ability ($v < 1$) do not enter the market. This is not only the privately optimal entry configuration but also the first best solution for this economy.

As in the previous section, consider first the case where the option to operate in the shadow economy is not available. Given the official's demands m for the license, the marginal type who is indifferent between entry and exit is given by $(v-1)K - m = 0$, that is, $v = m / K + 1$. The entry behavior of the entrepreneurs implies that the corrupt official maximizes his revenue:

[16] Rauch (1991) defines size dualism as the existence of a difference in size between the smallest formal sector firm and the largest informal sector firm.

[17] We need to impose an upper bound K on the level of capital employment since we assume a linear production technology in k. If we instead assume that the entrepreneur's revenue in the official sector is concave in k, that is, $vg(k)$, where $g'>0$ and $g''<0$, we will always have an interior solution and can dispense with the assumption of an upper bound on capital.

$$\max_{m} \; R(m) = m \cdot F\left(\frac{m}{K}+1\right).$$

Since the official's demand schedule *m* is uniquely determined by *v*, we treat *v* as the control variable:

$$\max_{v} \; R(v) = K(v-1)\cdot F(v).$$

The marginal entrant v^* that maximizes the corrupt official's revenue is implicitly given by the first order condition:

$$F(v^*) + (v^*-1)\cdot F'(v^*) = 0. \tag{6}$$

The number of entrants is given by $F(v^*)$. The official demands $m^* = (v^*-1)K$ for the license. Since $F(v)$ is the *inverse* cumulative distribution function and $F'(v) \leq 0$, the first order condition (6) implies that v>1. Thus, in our extended model with endogeneous investment, corruption induces exit of entrepreneurs whose types belong to $[v^{FB} = 1,\ v^*)$.

In the presence of the shadow economy, assume further that the probability of detection for an entrepreneur is $\mu \cdot k$ where μ again measures the monitoring effort of the supervising institution. The detection probability now also depends on the employed capital *k*, the idea being that the detection probability rises with the size of the undertaking as it becomes more and more difficult to hide the entrepreneurial activity from the supervisor. The entrepreneur's profit in the shadow economy is then given by $\pi_{SE} = (1-\mu\cdot k)\cdot v\cdot k - k$. As the probability of detection depends on the amount of capital employed in a firm, the entrepreneur who is active in the shadow economy has to trade off the potentially larger profit from a larger capital stock with the higher risk of detection:

$$\max_{k} \; \pi_{SE} = (1-\mu\cdot k)\cdot v\cdot k - k$$

which yields

$$\hat{k}_{SE}(v) = \frac{1}{2\cdot\mu}\cdot\left(1-\frac{1}{v}\right).$$

Thus, the optimal level of capital in the shadow economy, $\hat{k}_{SE}(v)$, is inversely related to the level of monitoring (μ) and positively related to the entrepreneur's ability (*v*). Since there is a constraint on the maximum amount of capital (K) that can be employed, the choice of capital for type v entrepreneur in the shadow economy is given by

$$k_{SE} = \min\left[\hat{k}_{SE}(v), K\right] = \min\left[\frac{1}{2\cdot\mu}\cdot\left(1-\frac{1}{v}\right), K\right].$$

Let $\tilde{v}$ again denote the marginal type of entrepreneur who is indifferent between entering the shadow and the official economy. Then, we can consider two cases depending on whether the constraint on the maximum amount of capital (K) is binding or not for the optimal level of capital employed in the shadow economy (see Figure 2).

<u>Case I:</u> $k_{SE}(v) = \hat{k}_{SE}(v) < K$

Let us first consider the case where the monitoring effort μ is sufficiently large so that the capital input in the shadow economy is distorted, i.e. $\hat{k}_{SE}(v) < K$. The entrepreneur's profit in the shadow economy becomes

$$\pi_{SE} = \frac{1}{\mu}\cdot\frac{(v-1)^2}{4\cdot v}.$$

Rather than entering the shadow economy, the entrepreneur has two further options. First, he can stay out of business, which earns him zero profits ($\pi_0 = 0$). Second, he can enter the official economy with the maximum firm size K. The profit earned in the official economy amounts to $\pi_{OE} = (v-1)\cdot K - m$.

Given m, the marginal investor is just indifferent to whether he enters the official economy or the shadow economy:

$$\pi_{OE} = \pi_{SE} \qquad \Leftrightarrow \qquad (v-1)\cdot K - m = \frac{1}{\mu}\cdot\frac{(v-1)^2}{4\cdot v}.$$

The corrupt official takes into account that firms might circumvent his discretionary power over licenses by producing in the shadow economy and maximizes

$$\max_{v}\ \tilde{R} \equiv m(v)\cdot F(v) = \left[(v-1)\cdot K - \frac{1}{\mu}\cdot\frac{(v-1)^2}{4\cdot v}\right]\cdot F(v),$$

which yields

$$\frac{\partial \widetilde{R}}{\partial v} = \left[F(v) + (v-1)\cdot F'(v)\right]\cdot\left[K - \frac{1}{4\cdot\mu}\cdot\frac{v-1}{v}\right] - \frac{1}{4\cdot\mu}\cdot F(v)\cdot\frac{v-1}{v^2} = 0\,.\text{[18]} \tag{7}$$

Then, $\widetilde{v}$, the marginal type of entrepreneur entering the official economy, is the solution to $\partial\widetilde{R}/\partial v = 0$. As in the basic model, entrepreneurs with low abilities ($v < 1$) do not enter at all. A second group of entrepreneurs with intermediate abilities ($1 \le v < \widetilde{v}$) operates in the shadow economy and runs inefficiently small firms. Those with high abilities ($\widetilde{v} \le v$) make the contribution to the corrupt official and operate legally with capital *K*.

Case II: $k_{SE}(v) = K$

Suppose now that the capital constraint is binding and at least the marginal type (the highest type entrepreneur who enters the shadow economy) installs the maximum capital. The marginal investor is then given by

$$\pi_{OE} = \pi_{SE} \qquad \Leftrightarrow \qquad v = \frac{m}{\mu\cdot K^2}\,.$$

The corrupt official maximizes his revenue

$$\max_{v}\ \widetilde{R} \equiv m(v)\cdot F(v) = \mu\cdot K^2\cdot v\cdot F(v)\,,$$

which yields

$$\partial\widetilde{R}/\partial v = \mu\cdot K^2\cdot[F(v) + v\cdot F'(v)] = 0 \tag{8}$$

as the first order condition which implicitly defines the optimal license payment.

Corruption creates incentives for entrepreneurs to run their businesses on an inefficiently small scale in the shadow economy. Despite this distortion, the message from our basic model still goes through.

Proposition 2. In an economy with corruption, welfare is higher when production in the unofficial sector is feasible, even though (at least some) firms in the shadow economy are inefficiently small.

[18] Once again, our assumption that the distribution of entrepreneurs' types satisfies the monotone hazard rate condition ensures that the second order condition for the maximization problem is satisfied.

Proof. (a) Case I: we evaluate (7) at v^*, which is the marginal type of entrepreneur when no unofficial sector exists. The first term in (7) becomes zero [cf. (6)] and we have $\partial\widetilde{R}(v^*)/\partial v < 0$. Hence, $\widetilde{v} < v^*$. (b) Case II: Evaluating (8) at v^* immediately shows that $\partial\widetilde{R}(v^*)/\partial v < 0$. Hence, $\widetilde{v} < v^*$ also holds in this case. When production in the shadow economy is feasible, production in the official sector is larger, i.e. $\widetilde{v} < v^*$, and there is some production in the shadow economy. Hence, overall welfare is higher with the existence of an unofficial sector.

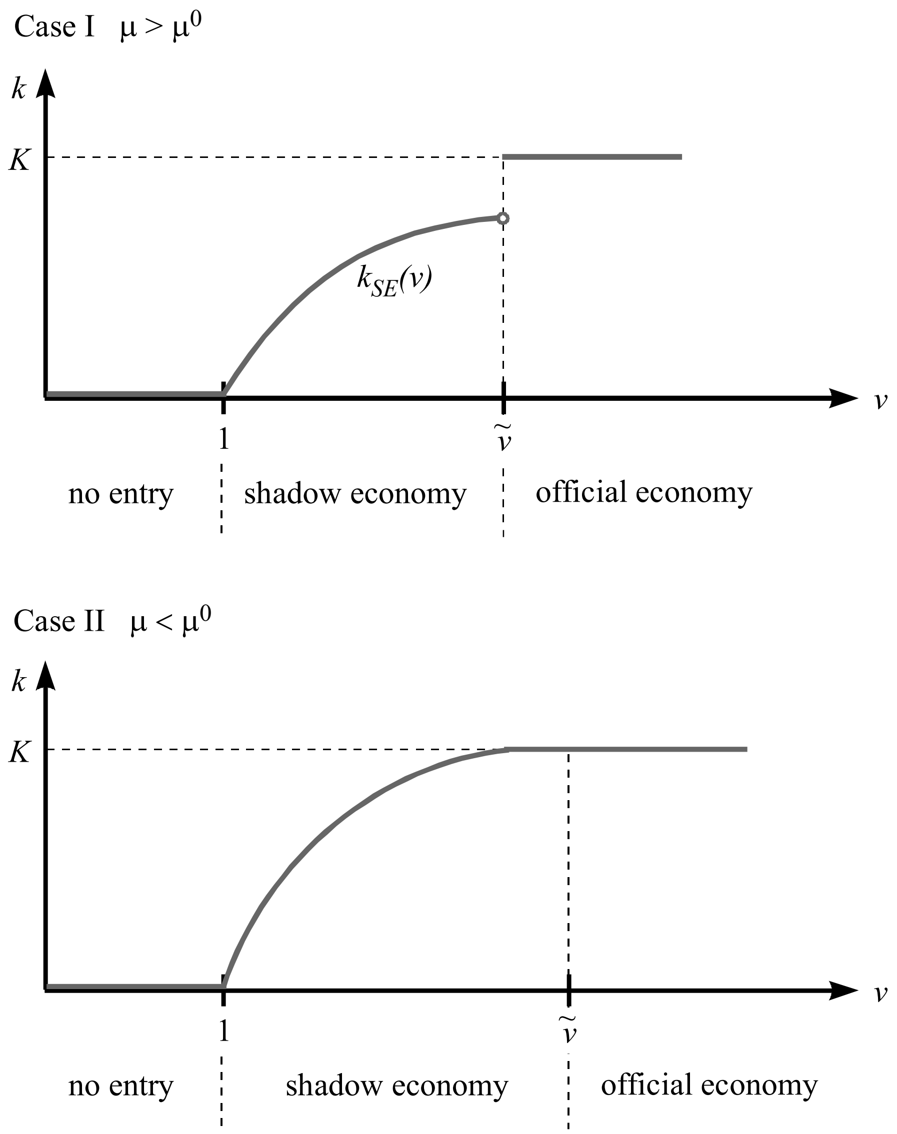

Figure 2.

The next proposition shows that the monitoring rate μ decides which case prevails.

Proposition 3. There is a critical level of the monitoring rate μ^0, such that if $\mu > \mu^0$ we have Case I ($k_{SE} < K$), and if $\mu \le \mu^0$ we have Case II ($k_{SE} = K$). The critical level of monitoring rate is $\mu^0 = \frac{1}{2K} \cdot \left(1 - \frac{1}{v^0}\right)$, where v^0 is defined by $F(v^0) + v^0 \cdot F'(v^0) = 0$.

Proof. See the Appendix.

From this point on, we will focus on Case I where $k_{SE} < K$, i.e., the optimal capital choice by entrepreneurs in the shadow economy is inefficiently small, since Case I generates more interesting and realistic results relevant to the shadow economy. In addition, if we dispensed with the assumption of the upper bound on the capital level and instead assumed that the revenue from employing capital level *k* is $v \cdot g(k)$ with $g'(k) > 0$ and $g''(k) < 0$ for type *v* entrepreneurs, we would always have Case I.

The discontinuity in the level of capital employment between the formal and informal sectors implies that there is firm size dualism in the economy in that the largest informal sector firm is smaller than the smallest formal sector firm. Rauch (1991) also derives such size dualism in his model of self-selection in which labor is divided into formal sector managers, formal sector employees, informal sector managers, and informal sector employees. In Rauch's model, there is labor market dualism in that a minimum wage that exceeds the competitive equilibrium wage is enforced only for firms larger than a certain size. Thus, he assumes a discontinuity in the enforcement activities to generate a discontinuity in the size distribution of firms. In our model, we were able to derive the discontinuity in the size distribution of firms even though the detection probability is continuous in the firm size represented by *k*.

Comparative Statics Results

As market entry is costly due to the existence of corrupt officials, entrepreneurs would like to flee the official economy. What prevents the entrepreneurs from simply switching to the informal sector is the threat of detection and punishment. So far the detection probability was exogenously given. To analyze the consequences of an improvement in the monitoring technology on the shadow economy, we parameterize the cost function of monitoring as $c(\mu,\alpha)$, where α measures the efficiency of the monitoring technology with $\partial c/\partial\alpha < 0$ and $\partial^2 c/(\partial\mu\partial\alpha) < 0$. The official maximizes his revenue net of the monitoring costs

$$\max_{v,\mu}\ \widetilde{P} \equiv \widetilde{R}(v) - c(\mu,\alpha).$$

In Case I of the previous analysis[19], the first-order conditions are given by

$$\frac{\partial\widetilde{P}}{\partial v} = 0 \quad\Leftrightarrow\quad [F(v)+(v-1)\cdot F'(v)]\cdot\left[K - \frac{1}{4\cdot\mu}\cdot\frac{v-1}{v}\right] - \frac{1}{4\cdot\mu}\cdot F(v)\cdot\frac{v-1}{v^2} = 0$$

and

$$\frac{\partial\widetilde{P}}{\partial\mu} = 0 \quad\Leftrightarrow\quad \frac{1}{\mu^2}\cdot\frac{(v-1)^2}{4\cdot v}\cdot F(v) - \frac{\partial c}{\partial\mu} = 0.$$

By totally differentiating the first-order conditions, we get

$$\begin{bmatrix} \dfrac{\partial^2\widetilde{P}}{\partial v^2} & \dfrac{\partial^2\widetilde{P}}{\partial v\partial\mu} \\[2ex] \dfrac{\partial^2\widetilde{P}}{\partial\mu\partial v} & \dfrac{\partial^2\widetilde{P}}{\partial\mu^2} \end{bmatrix} \begin{bmatrix} \dfrac{dv}{d\alpha} \\[2ex] \dfrac{d\mu}{d\alpha} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial^2\widetilde{P}}{\partial v\partial\alpha} \\[2ex] -\dfrac{\partial^2\widetilde{P}}{\partial\mu\partial\alpha} \end{bmatrix}.$$

By using Cramer's rule, we have

$$\frac{dv}{d\alpha} = \frac{\begin{vmatrix} -\dfrac{\partial^2\widetilde{P}}{\partial v\partial\alpha} & \dfrac{\partial^2\widetilde{P}}{\partial v\partial\mu} \\[2ex] -\dfrac{\partial^2\widetilde{P}}{\partial\mu\partial\alpha} & \dfrac{\partial^2\widetilde{P}}{\partial\mu^2} \end{vmatrix}}{H} \quad\text{and}\quad \frac{d\mu}{d\alpha} = \frac{\begin{vmatrix} \dfrac{\partial^2\widetilde{P}}{\partial v^2} & -\dfrac{\partial^2\widetilde{P}}{\partial v\partial\alpha} \\[2ex] \dfrac{\partial^2\widetilde{P}}{\partial\mu\partial v} & -\dfrac{\partial^2\widetilde{P}}{\partial\mu\partial\alpha} \end{vmatrix}}{H}$$

where $H = \partial^2\widetilde{P}/\partial v^2 \cdot \partial^2\widetilde{P}/\partial\mu^2 - \left(\partial^2\widetilde{P}/(\partial v\partial\mu)\right)^2$ is the determinant of the Hessian matrix with $H > 0$ by the second-order condition for maximization. It can easily be verified that

$$\frac{\partial^2\widetilde{P}}{\partial v\partial\alpha} = 0,\ \frac{\partial^2\widetilde{P}}{\partial\mu\partial\alpha} = -\frac{\partial^2 c}{\partial\mu\partial\alpha} > 0 \text{ and } \frac{\partial^2\widetilde{P}}{\partial v\partial\mu} > 0.$$

[19] The analysis for Case II is straightforward.

Therefore, we can sign the expressions above:

$$sign\left(\frac{dv}{d\alpha}\right) = sign\left(\frac{\partial^2 \widetilde{P}}{\partial\mu\partial\alpha} \cdot \frac{\partial^2 \widetilde{P}}{\partial v \partial\mu}\right) > 0 \qquad \text{and}$$

$$sign\left(\frac{d\mu}{d\alpha}\right) = sign\left(-\frac{\partial^2 \widetilde{P}}{\partial v^2} \cdot \frac{\partial^2 \widetilde{P}}{\partial\mu\partial\alpha}\right) > 0 .$$

Proposition 4. An increase in the efficiency of the monitoring technology ($\alpha\uparrow$) induces a higher rate of monitoring/detection by the official ($\mu\uparrow$). The better control over entrepreneurs allows the corrupt official to charge a higher bribery payment *m*. This drives additional entrepreneurs into the shadow economy, and the number of firms in the official economy shrinks ($v\uparrow$).[20]

The prevalence of the shadow economy is often associated with lax enforcement by the government. Proposition 4, however, demonstrates that the relationship between the size of the shadow economy and the monitoring efforts is more complex than it appears. *Given* the level of corruption (represented by *m*), an increase in the efficiency of the monitoring technology induces more monitoring efforts and consequently makes participation in the shadow economy relatively less attractive compared to participation in the official economy. The level of corruption by the official, however, is an *endogenous* variable that depends on the efficiency of the monitoring technology. With more efficient monitoring technology, the official is less worried about the possibility of defection to the shadow economy and is able to charge a higher license fee for entry.

According to Proposition 4, the direct effect of an increase in the efficiency of the monitoring technology is dominated by the indirect effect through *m*. Thus, the size of the official economy paradoxically shrinks as a result of more efficient monitoring technology. The overall effects of a more efficient monitoring technology on the size of the shadow economy are less clear. The reason is that even though the number of entrepreneurs in the

[20] The comparative statics were carried out for Case I. For Case II, the only difference is that the size of the official economy remains constant when the monitoring technology improves (dv/da=0).

shadow economy increases, they now operate on smaller scales than before. It is possible that overall outputs in the shadow economy are reduced even though more entrepreneurs are in the unofficial sector. This implies that we might obtain different answers to the size of the unofficial economy depending on how we measure the unofficial sector.

4. Public Goods

Most models analyzing corruption simply assume the existence of corrupt officials having discretionary power over the entry of firms.[21] In the background, however, there must be some useful (but not explicitly modeled) purpose for the existence of the public officials. Otherwise, the simple solution to the corruption problem would be to deregulate the economy, to abolish licenses and to get rid of the potentially corrupt officials. Hence, there must be some kind of market failure in the first place that is corrected by imposing regulations, installing a bureaucracy and requiring licenses for firms.

We consider now an extension of our basic model where the public official not only issues licenses but also provides a public good that is used in production. In this case, an increase in the shadow economy may lead to decreased state revenue, which in turn may reduce the quality and quantity of public good that enhances the productivity of entrepreneurs. For simplicity of the analysis, we assume that the required capital for entry k is fixed as in section 2. Let θ measure the amount of public good that is provided at cost $C(\theta)$ [with $C'(\theta) > 0,\ C''(\theta) > 0$] by the official. The public good enters positively into the production; the productivity of an entrepreneur of type v is $\theta \cdot v$.

Corruption without the Shadow Economy

What are the incentives of a corrupt official to provide the public good? To answer this question, we start with the scenario where firms face a corrupt official but do not have the opportunity to escape into the shadow economy. Hence, the entrepreneurs' only decision is whether to enter or not. Entry requires the bribery payment m to the corrupt official and the fixed investment k. Entrepreneurs with non-negative net revenues enter the market:

[21] An exception is Acemoglu and Verdier (2000).

$\theta \cdot v - m - k \geq 0$. Given the entry decision of firms, the official uses the bribery demand and the provision of the public good to maximize his revenues:[22]

$$\max_{v,\theta} \ R \equiv (\theta \cdot v - k) \cdot F(v) - C(\theta).$$

The first order conditions

$$\frac{\partial R}{\partial v} = \theta \cdot F(v) + (\theta \cdot v - k) \cdot F'(v) = 0 \tag{9}$$

$$\frac{\partial R}{\partial \theta} = v \cdot F(v) - C'(\theta) = 0 \tag{10}$$

determine entry v^* and the amount of the public good θ^*.

Proposition 5. Given the number of entrepreneurs $F(v^*)$, the level of public good provision is sub-optimally low.

Proof. Given the number of entrepreneurs $F(v^*)$, i.e. all entrepreneurs of type $v \geq v^*$ entering, the optimal provision of the public good can be found by solving

$$\max_{\theta} \ -\int_{v^*}^{\infty} \theta \cdot v \cdot F'(v)dv - C(\theta).$$

The first order condition

$$-\int_{v^*}^{\infty} v \cdot F'(v)dv - C'(\theta^{opt}) = 0$$

determines the socially optimal level of the public good θ^{opt} (Samuelson rule). Integration by parts of the first term on the left hand side shows $-\int_{v^*}^{\infty} v \cdot F'(v)dv \geq v^* \cdot F(v^*)$. As we have $v^* \cdot F(v^*) = c'(\theta^*)$ from (10), marginal benefits and marginal costs of the public good when provided by the corrupt official are below the socially optimal level: $C'(\theta^{opt}) \geq C'(\theta^*)$. Therefore, the level of public good provision is sub-optimally low: $\theta^{opt} \geq \theta^*$.

The intuition for this result is the following. The choice of θ^* by the corrupt official is determined by the marginal type *v**. An increase in the benefit of the marginal entrant is captured via higher bribery demands *m* by the corrupt official. The effect on the inframarginal entrants is irrelevant for the official as he cannot price discriminate between firms. In contrast,

[22] Again, we use *v* as a control instead of *m*.

the second-best level θ^{opt} is determined by the aggregate (or average) benefits for all entrants with types $[v^*, \infty)$. As the average type is more productive than the marginal type, the second-best level for the public good exceeds the level provided by the corrupt official.[23]

Corruption with the Shadow Economy

When the government provides public goods as an input for firms, the evasion into the shadow economy comes at a cost for the entrepreneurs. They do not take the risk of detection and punishment but they also forego some of the benefits created by the public good. We assume that the entrepreneurs operating in the shadow economy have a productivity of $\alpha \cdot \theta \cdot v$ where α, $0 \leq \alpha \leq 1$, denotes the extent to which the official can exclude the use of the public good by those who are in the shadow economy. For instance, $\alpha = 1$ implies that there is no exclusion; firms in the shadow economy benefit equally from public goods. With $\alpha = 0$, complete exclusion is feasible and production in the shadow economy becomes impossible.[24]

Entrepreneurs compare the payoffs from entering the official economy [$\theta \cdot v - m - k$] or entering the shadow economy [$(1-\mu) \cdot \alpha \cdot \theta \cdot v - k$]. Given the entry behavior of firms, the corrupt official maximizes his revenues:

$$\max_{v,\theta} \ \tilde{R} \equiv \left[1 - (1-\mu) \cdot \alpha\right] \cdot \theta \cdot v \cdot F(v) - c(\theta).$$

The first order conditions

$$\frac{\partial \tilde{R}}{\partial v} = F(v) + v \cdot F'(v) = 0 \tag{11}$$

$$\frac{\partial \tilde{R}}{\partial \theta} = \left[1 - (1-\mu) \cdot \alpha\right] \cdot v \cdot F(v) - c'(\theta) = 0 \tag{12}$$

again determine entry $\tilde{v}$ and the amount of the public good $\tilde{\theta}$. How does the existence of the shadow economy as an outside option for entrepreneurs affect the outcome?

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[23] This point is closely related to a monopolist's choice on product quality; see Spence (1975) and Tirole (1988, pp. 100-102).

[24] The precise value of α will depend on the type of public goods provided by the government. If the public good provided is an infrastructure such as roads and bridges that can be used by everyone, the appropriate value for α would be 1. In the case of government loan guarantees and education programs targeted towards registered firms, the relevant α would be zero. Law and order correspond to the case where α is somewhere between zero and one. A higher level of law and order would contribute to the general business activities in both sectors. However, entrepreneurs in the unofficial sector would have recourse to the law enforcement agency, say, in the case of breach of contracts.

First, the existence of the shadow economy once again induces more entrants. Evaluating (11) at $v = v^*$ yields $F(v^*) + v^* \cdot F'(v^*) = k \cdot F'(v^*)/\theta < 0$ [cf. (9)]. Hence, the result from our basic model also holds for the scenario with public goods: $\tilde{v} < v^*$.

Second, the existence of the shadow economy leads to a further underprovision of the public good. Evaluating (12) at $v = v^*$ yields $-(1-\mu) \cdot \alpha \cdot v \cdot F(v) < 0$ and therefore $\tilde{\theta} < \theta^*$. As we already know from Proposition 5, the decision on the public good by a corrupt official already leads to an underprovision ($\theta^* < \theta^{opt}$). Hence, the existence of the unofficial sector aggravates the inefficient provision of the public good.[25]

Including the provision of public goods explicitly into our model still confirms our result from the basic model, namely that, in the presence of corruption, the unofficial sector acts as a complement rather than substitute to the formal economy. The entrepreneurs' opportunity to move production to the shadow economy mitigates the corrupt official's extortive power and leads to more economic activity even in the formal economy. With public good provision, however, there is also a downside to the leeway created by the shadow economy. The increased flexibility of entrepreneurs also reduces the incentive for corrupt officials to provide the public good and leads to a further underprovision relative to the second-best optimum.

The empirical results on the shadow economy's effects on the official economy are ambiguous. Some studies show that a growing shadow economy has a negative impact on official GDP growth whereas other studies show the opposite effect. The model in this section may provide a clue to explaining the reason for the ambiguous empirical results concerning the effect of the shadow economy on the official economy.

5. Concluding Remarks

In this paper, we develop a simple framework to analyze the links between corruption and the shadow economy and their implications for the official economy. In a model of self-selection with heterogeneous entrepreneurs, we show that the entrepreneurs' option to flee to the

[25] This result is in line with the findings in the model by Johnson, Kaufmann and Shleifer (1997).

underground economy constrains the corrupt official's ability to introduce distortions to the economy for private gains. The unofficial economy thus mitigates government-induced distortions and as a result, leads to enhanced economic activities in the official sector. In this sense, the presence of the unofficial sector acts as a complement to the official economy rather than a substitute. This result is in sharp contrast to the existing models of unofficial economy where the official and unofficial sectors compete for resources and the existence of the informal sector is viewed as harmful for economic growth [Loayza (1996) and Johnson, Kaufmann and Shleifer (1997)].

The shadow economy is often considered as a debilitating force that saps the official economy by attracting factors of production away from the official economy and creating competition for official firms. As such, most countries attempt to control underground economic activities through various punitive measures [Schneider and Enste, 2000]. However, when corruption is defined as "the abuse of public power for private benefit" and the avoidance of it is the raison d'être of the shadow economy, any efforts to eradicate the shadow economy without tackling the principal problem of corruption would be counterproductive. Our model thus suggests the importance of considering the genesis of the shadow economy to evaluate implications of the shadow economy for resource allocations.

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Appendix

Proof of Proposition 3

Let $\bar{v}$ be defined by

$$k_{SE}(\bar{v}) = \frac{1}{2\cdot\mu}\cdot\left(1-\frac{1}{\bar{v}}\right) = K .$$

That is,

$$\bar{v} = \frac{1}{1-2\cdot\mu\cdot K}.$$

Since $k_{SE}(v)$ is increasing in v, whether the largest firm in the shadow economy is smaller than the efficient size (K) and whether there is a discontinuity in the scale of operation between the formal and informal sectors depends on the relative magnitude of $\bar{v}$ and $\tilde{v}$. If $\tilde{v} < \bar{v}$, Case I prevails, i.e., $k_{SE}(\tilde{v}) < K$, and if $\tilde{v} > \bar{v}$, Case II prevails, i.e. $k_{SE}(\tilde{v}) = K$.

Note that in Case II, $\tilde{v} = v^0$ where v^0 satisfies

$$F(v^0) + v^0 \cdot F'(v^0) = 0 \qquad \text{[cf. (7)].}$$

For $\tilde{v} = v^0$ to be the marginal type, we must have

$$\bar{v} = \frac{1}{1-2\cdot\mu\cdot K} \leq \tilde{v} = v^0 .$$

The condition for this to hold is that

$$\mu \leq \mu^0 = \frac{1}{2\cdot K}\cdot\left(1-\frac{1}{v^0}\right).$$

We also note that the first-order condition (6) for Case I when evaluated at $\bar{v}$ becomes after routine manipulation

$$\left.\frac{\partial\tilde{R}}{\partial v}\right|_{v=\bar{v}} = F(\bar{v}) + \bar{v}\cdot F'(\bar{v}).$$

If $\mu > \mu^0$, we have $\bar{v} = 1/(1-2\cdot\mu\cdot K) > v^0$ since v^0 is independent of μ and determined only by the distribution F. This implies that

$$\left.\frac{\partial \widetilde{R}}{\partial v}\right|_{v=\bar{v}} = F(\bar{v}) + \bar{v} \cdot F'(\bar{v}) < 0 .$$

Therefore, the optimal marginal type $\tilde{v}$ that satisfies the first-order condition (6) $\partial \widetilde{R} / \partial v = 0$ is lower than $\bar{v}$, that is $k_{SE}(\tilde{v}) < K$ if $\mu > \mu^0$.

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